

Quantum superconductor-metal transition

B.Spivak

Physics Department, University of Washington, Seattle, WA 98195

A.Zyuzin

A.F.Ioffe Physical- Technical Institute, 194021, St.Petersburg, Russia

M.Hruska

Physics Department, University of Washington, Seattle, WA 98195

Abstract

We consider a system of superconducting grains embedded in a normal metal. At zero temperature this system exhibits a quantum superconductor-normal metal phase transition. This transition can take place at arbitrary large conductance of the normal metal.

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I. INTRODUCTION

Quantum superconductor-insulator (or superconductor-normal metal) phase transitions can take place at zero temperature due to variation of parameters of the system. For example in experiments [1,3] the transition takes place as a function of the degree of disorder in a superconducting film. In experiments of [2,4,5] the transition was mediated by a magnetic field.

It has been suggested that a disordered superconducting film can be described as a network of Josephson-coupled superconducting grains shunted by resistors [6,7,9,11]. In this case the Coulomb interaction between electrons in grains suppresses fluctuations of the number of electrons in a grain and, due to uncertainty principle, increases the amplitude of fluctuations of the phase of the superconducting order parameter [7,9–11]. The competition between the charging energy and the Josephson inter-grain coupling energy leads to the phase transition. Shunting resistors play a double role in the model: a) dissipation in the resistors tends to suppress fluctuations of the phase of the order parameter b) tunneling between superconducting grains and the resistors renormalizes the capacitance of the grains and thereby the charging energy. As a result, in the framework of this model and in two dimensional case the onset of superconductivity corresponds to the normal state film's conductance G of the film of order of $\frac{e^2}{h}$. Since at $G < \frac{e^2}{h}$ the system should be in the insulating state, it has been conjectured that the transition is of the superconductor-insulator type [8–11] and that the behaviour of the system near the transition is universal. On the other hand, a renormalization group analysis [12], which starts from a perturbation theory for slightly disordered uniform superconductors, showed in 2D case a zero temperature superconductor-metal transition.

Recently Feigelman and Larkin [13] reconsidered the problem in the framework of a model of superconducting grains embedded in a normal metal film. They showed that a) the superconductor-normal metal transition takes place and b) deep in the metallic phase parameters of the system can be calculated with the help of perturbation theory. At $G > \frac{e^2}{h}$,

however, the critical concentration of grains turned out to be exponentially small.

On the other hand, the theories concerning granular superconductors [6,7,9,11] were done in the limit when the modulus of the order parameter on a grain does not fluctuate. In the case of no reflection on the superconductor-normal metal boundary this corresponds to $R \gg \xi$. Here R is the grain's radius, $\xi = \min[\frac{v_F}{\Delta_0}; \sqrt{\frac{D}{\Delta_0}}]$ is the zero temperature coherence length of the bulk superconductor, $D = \frac{v_F l}{3}$ is the electron diffusion coefficient in the metal, v_F is the Fermi velocity, l is the electron elastic mean free path and Δ_0 is the zero temperature value of the gap in the bulk superconductor.

In this paper we consider the opposite case when $R < \xi$ and show that the zero temperature quantum superconductor-metal transition as a function of grains' concentration can take place in samples with arbitrary large conductance and can exist even in the absence of any disorder in the sample and in the approximation when electrons in the normal metal do not interact. The critical concentration of the grains turns out to be relatively large.

We will consider a quasi-two-dimensional film of thickness $a \ll \xi$, which consists of superconducting grains of radius R embedded in a nonsuperconducting metal. The results can be easily generalized for d=3 case. We will assume that densities of states ν of the superconductor and the metal are the same and that the spatial dependence of the electron-electron interaction constant in the the sample has the form

$$\lambda(\mathbf{r}) = \begin{cases} \lambda_s > 0 & \text{if } |\mathbf{r} - \mathbf{r}_i| < R \\ \lambda_N < 0 & \text{if } |\mathbf{r} - \mathbf{r}_i| > R \end{cases} \quad (1)$$

Here λ_s and λ_N are electron interaction constants in the superconductor and in the metal respectively; \mathbf{r}_i are coordinates of centers of the superconducting grains and index i labels the grains. At zero temperature the linearized mean field equation [17] for the order parameter $\Delta(\mathbf{r})$

$$\Delta(\mathbf{r}) = \lambda(\mathbf{r}) \int d\mathbf{r}_1 K(\mathbf{r}, \mathbf{r}_1) \Delta(\mathbf{r}_1) \quad (2)$$

has a solution $\Delta(\mathbf{r}) = f(|\mathbf{r} - \mathbf{r}_i|)$ at $R = R_c^{(mf)}$. Assuming no reflection at the superconductor-metal boundary we get $R_c^{(mf)} \sim \xi$ [15]. Here $K(\mathbf{r}, \mathbf{r}_1) =$

$\int d\epsilon G(-\epsilon, \mathbf{r}, \mathbf{r}_1)G(\epsilon, \mathbf{r}, \mathbf{r}_1)$, $G(\mathbf{r}, \mathbf{r}_1)$ is the electron Green's function in the normal metal. It is convenient to normalize $\int d\mathbf{r}|f(\mathbf{r})|^2 = 1$. At $R < R_c^{(mf)}$ the mean field value of the order parameter is zero ($\Delta(\mathbf{r}) = 0$). To find the value of $\Delta(\mathbf{r})$ in the case $R > R_c^{(mf)}$ one has to add to Eq.2 terms nonlinear in Δ [17].

II. QUANTUM FLUCTUATIONS OF THE ORDER PARAMETER IN AN INDIVIDUAL GRAIN

We will consider the case $R < R_c^{(mf)}$ when for an individual grain Eq.2 does not have nonzero mean field solution. To calculate the correlation function of quantum fluctuations of the order parameter it is convenient to use a parametrization $\Delta_i(\mathbf{r}, t) = \alpha_i(t)f(|\mathbf{r} - \mathbf{r}_i|)$, which reflects the fact that, at $(R_c^{(mf)} - R) \ll R_c^{(mf)}$, the amplitude of quantum fluctuations of $\alpha_i(t)$ is large while the amplitude of fluctuations of the shape of $f(r)$ is small. To describe dynamics of the order parameter in a grain we use the effective action

$$S_i = \nu\tau_0 \int \frac{d\omega}{2\pi} (-i|\omega| + \frac{1}{\tau}) |\alpha_i(\omega)|^2 \quad (3)$$

whose derivation we outline in the Appendix. Here $\tau_0 = \min[\frac{r}{v_F}; \frac{R^2}{D}]$ is the time of electron flight through the grain and

$$\tau = \frac{\tau_0 R}{R_c^{(mf)} - R} \quad (4)$$

Using Eq.3 we get

$$\langle \alpha_i(\omega), \alpha_i^*(-\omega) \rangle_{(0)} = \frac{1}{\nu\tau_0(-i|\omega| + \frac{1}{\tau})} \quad (5)$$

which in t representation corresponds to

$$\langle \alpha_i(t), \alpha_i^*(0) \rangle_{(0)} = \begin{cases} \frac{1}{\nu\tau_0} (\frac{\tau}{t})^2 & \text{if } t \gg \tau \\ \frac{1}{\nu\tau_0} i[-i\pi + 2\ln(\frac{t}{\tau})] & \text{if } t \ll \tau \end{cases} \quad (6)$$

Here the subscript "(0)" indicates that the correlation function is calculated for a single existing grain "i". Eqs.5,6 hold as long as terms nonlinear in $|\alpha_i|^2$ in the effective action can be neglected, i.e. if $\tau \ll \delta^{-1}$. Here $\delta = (\nu R^2 a)^{-1}$ is the average

level spacing in the grain. They correspond to the casual Green's function $G_c = < T(\Psi_\sigma(\mathbf{r}, t)\Psi_{-\sigma}(\mathbf{r}, t)\Psi_\sigma^+(0, 0)\Psi_{-\sigma}^+(0, 0)) >$, where σ is the spin index. To get the retarded Green's function $G_R = < \theta(t)[\Psi_\sigma^+(\mathbf{r}, t)\Psi_{-\sigma}^+(\mathbf{r}, t), \Psi_\sigma(0, 0)\Psi_{-\sigma}(0, 0)]_+ >$ one has to make analytical continuation of Eq.5 with respect to ω and then to make a Fourier transform. As a result, $G_R(t) \sim \exp(-\frac{t}{\tau})$. It is interesting that the asymptotic time dependence of Eq.6 is the same as the one obtained in the case $R \gg R_c^{(mf)}$ [13] with the help of a complicated renormalization group analysis of the Caldeira-Leggett effective action [14]. In the latter case there is a non-zero mean field order parameter on a grain and the correlation function decays with time due to phase fluctuations mediated by the interaction with quantum electromagnetic fluctuations in conducting environment. At $G > \frac{e^2}{h}$, however, the corresponding correlation time turns out to be exponentially large, which is different from the case $R < R_c^{(mf)}$ Eq.4.

III. THE SUPERCONDUCTOR-METAL TRANSITION IN A SYSTEM OF SUPERCONDUCTING GRAINS

Let us consider now the case when the concentration of superconducting grains N embedded into the quasi-two-dimensional metallic film is finite. To describe this system in the case when α_i are small we will use the effective action $S = S_0 + S_{int}$ where $S_0 = \sum_i S_i$, while S_{int} describes inter-grain interaction via the normal metal and also the nonlinear in $|\alpha_i|^2$ contributions to the action.

$$S_{int} = - \sum_{i,j} \int d\omega J_{ij} \alpha_i(\omega) \alpha_j^*(-\omega) + b \sum_i \int d\omega_1 d\omega_2 d\omega_3 \alpha_i(\omega_1) \alpha_i^*(\omega_2) \alpha_i(\omega_3) \alpha_i^*(-\omega_1 - \omega_2 - \omega_3) \quad (7)$$

where J_{ij} have the meaning of Josephson coupling between grains i and j , and $b \sim \frac{R_c^2 \nu}{aD^2}$.

The mean field approximation corresponds to the minimum of S at $\omega = 0$. If $|\mathbf{r}_i - \mathbf{r}_j|$ is small enough the solution of the Uzadel equation Eq.25 yields Josephson coupling between two grains of the form ($|\mathbf{r}_i - \mathbf{r}_j| \gg R$)

$$J_{ij} = \frac{\nu R^2}{|\mathbf{r}_i - \mathbf{r}_j|^2} \quad (8)$$

However, in the case of finite two-dimensional concentration N such an expression for J_{ij} would lead to a logarithmic divergence of the ground state energy density.

On the other hand, at large $|\mathbf{r}_i - \mathbf{r}_j|$ electron-hole pairs diffusing through the metal between grains i and j will experience Andreev scattering from the superconducting grains situated between grains i and j . An example of such a grain "k" is shown in Fig.1. Due to the Andreev nature of reflection [16] electrons scattered by the grain "k" will be reflected into a hole moving in the direction opposite to the initial direction of electron's motion. Thus this electron will never reach the grain j . As a result we have

$$J_{ij} = \frac{\nu R^2}{|\mathbf{r}_i - \mathbf{r}_j|^2} \exp\left(-\frac{|\mathbf{r}_i - \mathbf{r}_j|}{L_0^{(mf)}}\right) \quad (9)$$

$$L_0^{(mf)} = \left(\frac{l}{NR} \frac{Ra^{1/2}\Delta_0}{\alpha_{(mf)}}\right)^{1/2} \quad (10)$$

Therefore the mean field equation for the ground state mean field value $\alpha_{(mf)}$ of the order parameter on a grain has a form

$$\frac{\nu\tau_0}{\tau} - 2\pi\nu R^2 N \ln(N^{1/2} L_0^{(mf)}) + b\alpha_{(mf)}^2 = 0 \quad (11)$$

Thus at $T = 0$ an exponentially small mean field solution for the order parameter

$$\alpha_{(mf)} \sim \Delta_0 l a^{1/2} \exp\left(-\frac{\tau_0}{\tau R^2 N}\right) \quad (12)$$

exists at arbitrary small N .

However, at small enough N the amplitude of quantum fluctuations of the order parameter Eqs.5,6 becomes larger than its mean field value. In this case the mean field theory is not applicable. To show that at small N quantum fluctuations destroy superconductivity completely and that the normal metal state is stable we use a perturbation theory procedure similar to the one used in [13]. A requirement for the perturbation theory in terms of J_{ij} to be valid is the convergence of the integral

$$\int < \alpha_i(t) \alpha_i^*(0) > dt < \infty \quad (13)$$

which in our case follows from Eq.6. The integral in this case equals $\frac{\tau}{\nu\tau_0}$.

Again, the Josephson couplings of the form Eq.8 would lead to divergency of the perturbation theory. To cut off the divergence in the absence of the magnetic field one can consider the case [13] when there is a repulsion between electrons in the metal and $\lambda_N \neq 0$. Then in the two-dimensional case we have $J_{ij} \sim \nu R^2 r^{-2} [1 + 2\nu|\lambda_N| \ln(\frac{r}{R})]^{-2}$ and the perturbation theory will converge on the lengthscale $L_\lambda \sim R \exp(\frac{1}{\nu|\lambda_N|})$. In the presence of a weak magnetic field H the Josephson intergrain coupling decays exponentially on distances larger than the magnetic length $L_H = \sqrt{\frac{m}{eH}}$. Thus the cut off length relevant for the convergence of the perturbation theory with respect to the term in Eq.7 proportional to J_{ij} is $L_0 = \min[L_\lambda; L_H]$. It gives small corrections to the Eqs.5, 6 as long as $NR^2 \ln(L_0 N^{1/2}) \ll \frac{\tau_0}{\tau}$. In the opposite case the ground state of the system is superconducting. Thus we can estimate a relation between the critical concentration of the grains N_c and their critical radius R_c from the equation

$$\frac{|R_c^{(mf)} - R_c|}{R_c^{(mf)}} \sim R_c^2 N_c \ln(L_0 N_c^{1/2}) \quad (14)$$

For example, in the case $R < R_c^{(mf)}$ and at $H = 0$ we have the estimate

$$N_c \sim \frac{1}{R_c^2} \nu |\lambda_N| \quad (15)$$

This is different from the case $R \gg R_c^{(mf)}$ where at $G > \frac{e^2}{h}$ the critical concentration is exponentially small. The difference originates from the difference in correlation times.

Neglecting the second term in Eq.7 and assuming for simplicity that superconducting grains form a square lattice we get an expression for the correlation functions

$$\begin{aligned} < \alpha_k(\omega) \alpha_l^*(-\omega) > &= \frac{1}{\nu\tau_0} (-i|\omega| + \frac{1}{\tau} + \frac{1}{\nu\tau_0} J_{ij})_{k,l}^{-1} \\ &= \frac{1}{\nu\tau_0} \sum_{\mathbf{q}} \exp(-i\mathbf{q}\mathbf{r}) \frac{1}{-i|\omega| + \frac{1}{\tau} - K(\mathbf{q})} \end{aligned} \quad (16)$$

where components of \mathbf{q} are whole multiples of $\frac{2\pi}{L}$ and L is the sample size.

$$K(q) = \frac{1}{\nu\tau_0} \sum_i \exp(i\mathbf{q}\mathbf{r}) K(\mathbf{r}_i) \quad (17)$$

At $|\mathbf{q}|N^{-1/2} \ll 1$ we have

$$K(\mathbf{q}) = -\pi D N \ln((1 + (qL_0)^2)L_0^2 N) \quad (18)$$

The mean field Eqs.16-18 are valid because the characteristic radius of interaction between grains $N^{-1/2} \ln(L_0 N^{\frac{1}{2}}) \gg N^{-1/2}$ is much larger than the average intergrain distance.

Qualitatively the picture of quantum superconducting fluctuations in the normal metal is similar to superconducting fluctuations in uniform metals at temperatures which are close to the critical one [18,19]:

a) Due to quantum fluctuations conductivity of the system is enhanced as compared to the normal metal value. It exhibits a big positive magnetoresistance.

b) At small magnetic field the zero temperature Hall coefficient is suppressed as compared to its normal metal value [22]. It also exhibits strong magnetic field dependence.

c) Diamagnetic susceptibility is enhanced and exhibits a strong nonlinearity as a function of magnetic field.

d) Energy dependence of the density of states at the Fermi surface has a dip, whose amplitude is magnetic field dependent.

IV. CONCLUSION

We have considered a model in which superconducting grains with the radius $R < R_c$ are embedded in the normal metal and have shown that in this case there is a zero temperature quantum superconductor-normal metal phase transition as a function of N and R . This transition is driven, primarily, by fluctuations of the modulus of the superconducting order parameter. Though the parameters of the transition, in principle, depend on D , it exists even in the case when there is no disorder in the sample. The critical concentration of grains turns out to be relatively large.

Calculations presented above did not take into account localization effects [20,21]. In d=3 case they are small. The question of whether in two-dimensional case the metallic state is localized [8] requires additional investigation. On the other hand, as we have discussed, the transition can take place at very large sample conductances when the localization length is also very large.

We would like to mention that in the presence of the electron-electron repulsion in the metal the metal-insulator transition can exist even in the mean field approximation described by the Eq.2. For example, in the case $\lambda_s \sim |\lambda_N|$ and $R < R_c^{mf}$ we have $N_c \approx \frac{1}{R^2}$.

Finally, we would like to mention, that in our opinion experimental data on 2-D films [1-5] do not contradict the possibility of a zero temperature superconductor-normal metal quantum phase transition.

V. APPENDIX

We start with a standard expression for the partition function in a superconductors (see for example [23]):

$$Z = \int D\Delta D\Delta^* \exp(iS_{eff}) \quad (19)$$

$$S_{eff} = \int d\mathbf{r} dt \left[\frac{|\Delta|^2}{\lambda(\mathbf{r})} + \int_0^1 d\eta Tr[\hat{\Delta}(\mathbf{r}, t) \hat{G}_\eta(\mathbf{r}t, \mathbf{r}t)] \right] \quad (20)$$

where $\hat{G}_\eta(\mathbf{r}t, \mathbf{r}'t')$ is a matrix Green function which is a solution of the Gorkov equation

$$\left[-i \frac{d}{dt} - \xi(i\nabla)\sigma_z + u(\mathbf{r}) - \eta \hat{\Delta}(\mathbf{r}, t) \right] \hat{G}_\eta(\mathbf{r}, t; \mathbf{r}'t') = \delta(\mathbf{r} - \mathbf{r}') \delta(t - t') \hat{I} \quad (21)$$

Here $u(\mathbf{r})$ is the external potential.

$$\hat{\Delta} = \begin{pmatrix} 0 & \Delta \\ \Delta^* & 0 \end{pmatrix} \quad (22)$$

and \hat{I} is a 2*2 unit matrix in Nambu space. We assume that $u(\mathbf{r})$ has a white noise statistics with correlation functions $\langle\langle u(\mathbf{r}) \rangle\rangle$ and $\langle\langle u(\mathbf{r})u(\mathbf{r}') \rangle\rangle = \frac{v_F}{l\nu} \delta(\mathbf{r} - \mathbf{r}')$. Here brackets $\langle\langle \rangle\rangle$ stand for averaging over realizations of the scattering potential.

Averaging Eq.21 over realizations of $u(\mathbf{r})$, neglecting all weak localization and mesoscopic corrections, and making the diffusion approximation we get Uzadel equations for the normal and the anomalous Green's functions (See, for example, [24])

$$\omega F(\omega, \mathbf{r}) + \frac{1}{2} D(F(\omega, \mathbf{r}) \nabla^2 G(\omega, \mathbf{r}) - G(\omega, \mathbf{r}) \nabla^2 F(\omega, \mathbf{r})) = \eta \Delta^*(\mathbf{r}) G(\omega, \mathbf{r}) \quad (23)$$

$$G^2 + |F|^2 = 1 \quad (24)$$

Expanding Eqs.20-24 with respect to Δ we get an expression for the effective action Eq.3 for an individual grain.

To get Eqs.9,10 we have to solve the Uzadel equation Eq.24 in the normal metal between the superconducting grains. This solution corresponds to an electron diffusion in the metal and Andreev reflections from the superconducting grains. In the case $R \gg R_c^{mf}$ we have $|\Delta(t)| = \Delta_0$ and the Andreev scattering cross section equals to R^2 . Since in our case $\Delta(t) \ll \Delta_0$ the cross section is of order of $R^2 \frac{\Delta^2(t)}{\Delta_0^2}$ and we get Eq.9,10.

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FIGURES

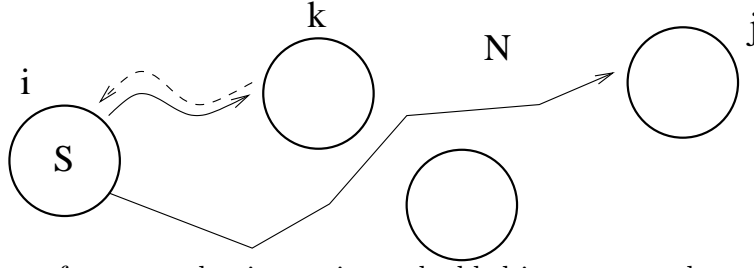


FIG. 1. A system of superconducting grains embedded into a normal metal. S stands for the superconductor while N stands for the normal metal. Solid lines correspond to trajectories of electrons diffusing between grains i and j . The dashed line corresponds to the hole reflected from the grain k .